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Particle Propagation in Cosmological Backgrounds

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Abstract We analyse the quantum propagation of interacting particles in cosmological backgrounds. The model we use consists of a doublet of massive scalar fields propagating in an expanding universe filled with massless radiation. The masses are much larger than the Hubble expansion rate, so that the number of massive particles is preserved and the fields adequately described within the adiabatic approximation. We focus on the dissipative effects related to the expansion rate by computing the imaginary part of the self-energy. In the quasi static approximation, we recover the expected result that instantaneous decay rate is governed by the local temperature. We then analyse the long time behavior of the propagator to unravel the secular effects induced by the self-energy. We show that these effects can be expressed in terms of integrals of local quantities. Applications to the trans-Planckian question are briefly discussed.

1 Introduction

In this paper we study the quantum propagation of particles in a cosmological background. We are particularly interested in understanding the dissipative phenomena related to the time dependence of the metric. To this end, we analyze the propagator of a massive particle

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which interacts with a massless radiation field in an expanding universe. We meet several difficulties.

First, we are dealing with an interacting field theory in a curved spacetime. In this situation, the asymptotic *in* and *out* vacua generally do not coincide. Being interested in expectation values, rather than in *in-out* matrix elements, we must adopt the Keldysh–Schwinger formalism, or Closed Time Path (CTP) method [13, 18, 29] in curved spacetime [11, 12, 17, 33].

Second, as it is well known, in a curved spacetime there is no single definition for the vacuum nor for the concept of particle. We overcome this problem by working within the adiabatic approximation ([7] and references therein): the massive particles will have their Compton wavelengths much smaller than the typical curvature radius of the universe (in our case, the Hubble radius). This approximation also has the virtue of greatly reducing the technical difficulties of the problem since one can obtain closed analytical expressions for the free propagators. Notice that, in general, in expanding universes, even the free propagators are not expressible in a closed analytic form.

Third, as explained in [2], in theories such as QED or perturbative quantum gravity, dissipative effects appear only at two loops, because the one-loop diagrams which could have lead to dissipation vanish on the mass shell. Here, in order to keep the calculations simple, we have chosen a simple, yet physically meaningful, model which exhibits dissipation at one loop. We expect QED or perturbative quantum gravity to have a similar behavior at two loops.

Finally, in flat spacetime it is common to extract the particle properties from the analytic structure of the (retarded) propagator. Such an interpretation is based, in the vacuum, on the spectral representation of the propagator (see, e.g., [9, 32]), or, in a thermal background (and in somewhat less rigorous grounds) on the linear response theory [15, 19]. In curved spacetime, there are some arguments pointing to a similar interpretation under the adiabatic approximation [1, 14], but we are not aware of a proof. While we shall not solve this question, we will show under which conditions the usual pole analysis of field theory can be extended to curved spacetime when working in the adiabatic approximation.

In this paper we compute the self-energy of the lightest field in a massive doublet which propagates in a thermal bath of massless particles in an expanding universe. We study how the aforementioned difficulties appear in this model, and how they can be overcome. We carry out the computation in one simple case, and postpone the generalization to other interesting cases for a later publication [4].

The paper is organized as follows. In Sect. 2 we introduce the model. In Sect. 3 we focus on the free propagators in a cosmological background. We motivate the use of the adiabatic approximation for the massive fields. In Sect. 4 we study the relation between the retarded propagator and the self-energy in cosmological backgrounds. In Sect. 5 we study the physical significance of the self-energy and its frequency representation in these backgrounds. Finally, in Sect. 6 we summarize the main points of the paper and discuss its relevance to the trans-Planckian question. Throughout this paper we use a system of natural units with $\hbar = c = 1$. The metric has the signature $(-,+,+,+)$.

2 The Model

We consider spatially isotropic and homogeneous Friedmann–Lemaître–Robertson–Walker models with flat spatial sections. The metric can be expressed as

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2, \tag{1a}$$

in physical time t, or as

$$ds^2 = a^2(\eta)(-d\eta^2 + d\mathbf{x}^2), \tag{1b}$$

in conformal time η, where $a\,d\eta = dt$. The scale factor a is arbitrary for the moment.

We introduce the following model: two massive fields ϕ_m, and ϕ_M, interacting with a massless field, χ, via a trilinear coupling. The total action is

$$S = S_m + S_M + S_\chi + S_{\text{int}}, \tag{2a}$$

$$S_m = \frac{1}{2}\int dt\, d^3\mathbf{x}\, a^3(t)\left((\partial_t\phi_m)^2 - \frac{1}{a^2(t)}(\partial_\mathbf{x}\phi_m)^2 - m^2\phi_m^2\right), \tag{2b}$$

$$S_M = \frac{1}{2}\int dt\, d^3\mathbf{x}\, a^3(t)\left((\partial_t\phi_M)^2 - \frac{1}{a^2(t)}(\partial_\mathbf{x}\phi_M)^2 - M^2\phi_M^2\right), \tag{2c}$$

$$S_\chi = \frac{1}{2}\int dt\, d^3\mathbf{x}\, a^3(t)\left((\partial_t\chi)^2 - \frac{1}{a^2(t)}(\partial_\mathbf{x}\chi)^2 - \xi R(t)\chi^2\right), \tag{2d}$$

$$S_{\text{int}} = gM\int dt\, d^3\mathbf{x}\, a^3(t)\phi_m\phi_M\chi, \tag{2e}$$

where $R(t)$ is the Ricci scalar. We assume that the massless field is conformally coupled to gravity, so that $\xi = 1/6$.

We consider the two massive fields with large masses but with a small mass difference $\Delta m := M - m \ll M$. As shown in [3], the model can be interpreted as a field-theory description of a relativistic two-level atom (of rest mass m and energy gap Δm) interacting with a scalar radiation field χ. This model was used in [24] to study the recoil effects of an accelerated two-level atom subject to the Unruh effect.

The radiation field χ is assumed to be at some conformal temperature θ. The corresponding physical temperature is chosen to be much smaller than the masses of the fields. We also choose the Hubble rate $H(t) := \dot{a}(t)/a(t)$ to be much smaller than the masses. These restrictions ensure that the number of massive particles is strictly conserved. The non-trivial dynamics concerns the transitions between the two massive fields accompanied by emission/absorption of massless quanta.

When studying the self-energy in these circumstances, it is useful to work with rescaled massive fields defined by, $\bar{\phi}(t,\mathbf{x}) := [-g(t,\mathbf{x})]^{1/4}\phi(t,\mathbf{x}) = a^{3/2}(t)\phi(t,\mathbf{x})$. In terms of these new fields the quadratic actions become

$$S_m = \frac{1}{2}\int dt\, d^3\mathbf{x}\left(a^3(t)\{\partial_t[a^{-3/2}(t)\bar{\phi}_m]\}^2 - \frac{1}{a^2(t)}(\partial_\mathbf{x}\bar{\phi}_m)^2 - m^2\bar{\phi}_m^2\right), \tag{3a}$$

and similarly for the massive field $\bar{\phi}_M$, whereas the interaction term becomes

$$S_{\text{int}} = gM\int dt\, d^3\mathbf{x}\,\bar{\phi}_m\bar{\phi}_M\chi. \tag{3b}$$

3 Free Propagators. The Adiabatic Approximation

In a curved spacetimes it is not a trivial task to compute free field vacuum propagators. On the one hand one needs a criterion to choose the initial (vacuum) state. On the other hand, even for simple cosmological models such as the ones we are considering, closed analytic expressions can only be found in some particular situations [7].

Since translation invariance along the spatial axes is preserved, we will study propagators of modes labeled by their comoving (and conserved) momentum $\mathbf{p}$.

3.1 Massless Fields

For massless conformally coupled fields there is a natural vacuum state, the conformal vacuum. Propagators in this vacuum, when expressed in conformal time, essentially correspond to the flat spacetime propagators. For instance, the retarded propagator is given by

$$\Delta_{\mathrm{R}}^{(0)}(\eta_1, \eta_2; \mathbf{p}) = \frac{-i}{a(\eta_1)a(\eta_2)p} \sin[(\eta_1 - \eta_2)p]\theta(\eta_1 - \eta_2), \tag{4}$$

where $p = |\mathbf{p}|$. In physical time the corresponding expression is simply:

$$\Delta_{\mathrm{R}}^{(0)}(t_1, t_2; \mathbf{p}) = \frac{-i}{a(t_1)a(t_2)p} \sin\left(\int_{t_2}^{t_2} \mathrm{d}t\, \frac{p}{a(t)}\right)\theta(t_1 - t_2). \tag{5}$$

All other propagators can be obtained in a completely analogous way. For instance, the positive Wightman function in a thermal bath is

$$\Delta_{+}^{(0)}(\eta_1, \eta_2; \mathbf{p}) = \frac{1}{a(\eta_1)a(\eta_2)2p}(e^{-i(\eta_1-\eta_2)p}[1 + n_\theta(p)] + e^{i(\eta_1-\eta_2)p}\, n_\theta(p)) \tag{6}$$

where $n_\theta(\varepsilon)$ is the Bose–Einstein function corresponding to a (constant) conformal temperature θ:

$$n_\theta(\varepsilon) := \frac{1}{e^{\varepsilon/\theta} - 1}. \tag{7}$$

3.2 Massive Fields

For the massive fields, rather than attempting to find the exact propagator, we will exploit the fact that their Compton wavelengths is much smaller than the Hubble length H^{-1}. In this regime, the adiabatic (WKB) approximation is valid.

Although the method is standard, we review the basic steps of the derivation applied to the retarded propagator because this will be useful in Sect. 5. The derivation of the expressions for the other propagators is completely analogous. The equation of motion of the retarded propagator $\bar{G}_{\mathrm{R}}^{(0)}(t_1, t_2; \mathbf{p})$ of the rescaled fields is

$$\left[\frac{1}{a^3(t)}\frac{\partial}{\partial t}\left(a^3(t)\frac{\partial}{\partial t}\right) + m^2 + \frac{p^2}{a^2(t)}\right]\left[\frac{\bar{G}_{\mathrm{R}}^{(0)}(t, t'; \mathbf{p})}{a^{3/2}(t)a^{3/2}(t')}\right] = \frac{-i}{a^3(t)}\delta(t - t'). \tag{8}$$

We introduce a new dimensionless time coordinate $u = ht$, where h is related to the Hubble parameter. The equation then reads:

$$\left[\frac{h^2}{a^3(u)}\frac{\partial}{\partial u}\left(a^3(u)\frac{\partial}{\partial u}\right) + m^2 + \frac{p^2}{a^2(u)}\right]\left[\frac{\bar{G}_{\mathrm{R}}^{(0)}(u, u'; \mathbf{p})}{a^{3/2}(u)a^{3/2}(u')}\right] = -\frac{ih}{a^3(u)}\delta(u - u'). \tag{9}$$

We choose the following WKB-like ansatz for the solution:

$$\bar{G}_{\mathrm{R}}^{(0)}(u,u';\mathbf{p}) = \frac{-i}{\sqrt{-\partial_u A(u,u';p)\partial_{u'}A(u,u';p)}}\sin\left(\frac{A(u,u';p)}{h}\right)\theta(u-u'), \tag{10}$$

for some function $A(u,u';p)$ to be determined. The singular part of the equation is automatically verified when $\partial_u A(u,u';p) = -\partial_{u'}A(u,u';p)$ in the limit $u \to u'$. As for the regular part we make a series expansion in h: $A = A_0 + hA_1 + h^2A_2 + \cdots$. The term in h^0 gives:

$$\left(\frac{\partial A_0(u,u';p)}{\partial u}\right)^2 = m^2 + \frac{p^2}{a(u)^2}. \tag{11}$$

The solution with the appropriate initial conditions is

$$A_0(u,u';p) = \int_{u'}^{u} \mathrm{d}u'' \sqrt{m^2 + \frac{p^2}{a^2(u'')}} = h\int_{t'}^{t} \mathrm{d}t'' \sqrt{m^2 + \frac{p^2}{a^2(t'')}}. \tag{12}$$

The next term A_1 vanishes, so that the first correction to the above expression in the series expansion of A is of order h^2/M^2. An evaluation of the first corrections confirms that they scale like H^2/M^2, where H is the characteristic value of the Hubble parameter in the interval $[t_1,t_2]$.

Therefore, up to terms in H^2/M^2, the retarded propagator for the massive fields is

$$\bar{G}_{\mathrm{R}}^{(0)}(t_1,t_2;\mathbf{p}) = \frac{-i}{\sqrt{E_{\mathbf{p}}(t_1)E_{\mathbf{p}}(t_2)}}\sin\left(\int_{t_2}^{t_1}\mathrm{d}t'\,E_{\mathbf{p}}(t')\right)\theta(t_1-t_2). \tag{13}$$

4 Interacting Propagators. The Self-Energy

The aim of this section is to compute the interacting retarded Green function

$$G_{\mathrm{R}}(t,t';\mathbf{p}) := \theta(t-t')\langle[\hat{\phi}_{m\mathbf{p}}(t),\hat{\phi}_{m\mathbf{p}}(t')]\rangle \tag{14}$$

within the adiabatic approximation. We will use perturbation theory within the CTP framework. We will not detail the aspects related to the CTP method; they can be found in [2, 3]. Instead, we concentrate on the novel aspects induced by the universe expansion.

The retarded Green function is (exactly) related to the retarded self-energy $\Sigma_{\mathrm{R}}(t,t';\mathbf{p})$ via

$$G_{\mathrm{R}}(t,t';\mathbf{p}) = G_{\mathrm{R}}^{(0)}(t,t';\mathbf{p}) - i\iint \mathrm{d}s\,\mathrm{d}s'\,\sqrt{-g(s)}\sqrt{-g(s')}G_{\mathrm{R}}^{(0)}(t,s;\mathbf{p})\Sigma_{\mathrm{R}}(s,s';\mathbf{p})G_{\mathrm{R}}(s',t';\mathbf{p}). \tag{15}$$

In terms of the rescaled fields, $\bar{\phi}(t;\mathbf{p}) = a^{3/2}\phi(t;\mathbf{p})$, the above relation becomes

$$\bar{G}_{\mathrm{R}}(t,t';\mathbf{p}) = \bar{G}_{\mathrm{R}}^{(0)}(t,t';\mathbf{p}) - i\iint \mathrm{d}s\,\mathrm{d}s'\,\bar{G}_{\mathrm{R}}^{(0)}(t,s;\mathbf{p})\bar{\Sigma}_{\mathrm{R}}(s,s';\mathbf{p})\bar{G}_{\mathrm{R}}(s',t';\mathbf{p}), \tag{16}$$

which is the same equation as in Minkowski spacetime. In other words $a(t)$ only enters through $\bar{\Sigma}$ and $\bar{G}_{\mathrm{R}}^{(0)}$.

We assume that the massive fields are in the adiabatic vacuum, and that the massless field χ is in a thermal state, characterized by a fixed conformal temperature θ. We will compute the one-loop self energy to order g^2 in the adiabatic approximation.

Additionally to this approximation, to further simplify the expressions, we will assume that the massive particles are slowly moving with respect to the cosmological frame, so that they can be treated non-relativistically. We will also discard the non-linear recoil effects introduced by the emission/absorption of massless particles, because they are of order $\Delta m/M$ and therefore can be neglected at the order of the approximation we are working.

Although we will apply Feynman rules in time space, the self-energy will be evaluated in the following frequency representation: we introduce the semisum and difference time coordinates, $T = (t_1 + t_2)/2$ and $\Delta = t_1 - t_2$, and then Fourier transform with respect to Δ. The usefulness of this representation will be discussed in next section. As for the spatial part, we work in the momentum representation to exploit conservation of the conformal momentum.

4.1 Imaginary Part of the Self-Energy

In the Closed Time Path formulation, the imaginary part[1] of the self-energy is given by

$$\operatorname{Im} \bar{\Sigma}_{\mathrm{R}}(t_1, t_2) = \frac{1}{2i}[\bar{\Sigma}^{12}(t_1, t_2) - \bar{\Sigma}^{21}(t_1, t_2)], \tag{17}$$

where indices refer to the different CTP branches (see e.g. (C18) in [2]). By applying the CTP Feynman rules in the physical time representation, and using the free propagators given in Sect. 3, to order g^2, we get

$$\begin{aligned}\bar{\Sigma}^{21}(t_1, t_2; \mathbf{p}) = {} & \frac{i g^2 M^2}{a(t_1)a(t_2)} \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \frac{1}{2\sqrt{E^*_{\mathbf{p}-\mathbf{k}}(t_1) E^*_{\mathbf{p}-\mathbf{k}}(t_2)}} e^{-i\int_{t_2}^{t_1} \mathrm{d}t'\, E^*_{\mathbf{p}-\mathbf{k}}(t')} \\ & \times \frac{1}{2|\mathbf{k}|}\big[e^{-i\int_{t_2}^{t_1} \mathrm{d}t' |\mathbf{k}|/a(t')}[1 + n_\theta(|\mathbf{k}|)] + e^{i\int_{t_2}^{t_1} \mathrm{d}t' |\mathbf{k}|/a(t')} n_\theta(|\mathbf{k}|)\big].\end{aligned} \tag{18}$$

with $E^{*2}_{\mathbf{p}-\mathbf{k}}(t) = (\mathbf{p}-\mathbf{k})^2/a^2(t) + M^2$. The two terms in (18) correspond to two Feynman diagrams showing respectively emission and absorption of a photon (the first diagram would vanish on shell in flat spacetime). The physical effects encoded in the self-energy are best extracted when working near the mass shell. To this end, we Fourier transform the above expression with respect to the time difference $\Delta = t_1 - t_2$, while maintaining the mid point $T = (t_1 + t_2)/2$ fixed:

$$\begin{aligned}\bar{\Sigma}^{21}(\omega, T; \mathbf{p}) &:= \int \mathrm{d}\Delta\; e^{i\omega\Delta}\, \bar{\Sigma}^{12}(t_1, t_2; \mathbf{p}) \\ &= -2i \operatorname{Im} \bar{\Sigma}_{\mathrm{R}}(\omega, T; \mathbf{p})\end{aligned} \tag{19}$$

where $t_1 = T + \Delta/2$ and $t_2 = T - \Delta/2$. The second equality is valid when $\omega \simeq m$ (near the mass shell), because the Fourier transform $\bar{\Sigma}^{12}$ gives exponentially damped contributions (in am/θ and m/H) in the high mass limit we using.

[1] Notice that here we are abusing the notation because in the time representation the retarded self-energy is purely real. "Imaginary part" here refers to the frequency representation we shall introduce next. In the time representation, it corresponds to the odd part of $\bar{\Sigma}_{\mathrm{R}}$ under the exchange of t_1 and t_2.

We now introduce the small-recoil and non-relativistic approximation, in which $E^*_{\mathbf{p}-\mathbf{k}}(t) \approx M + \mathbf{p}^2/[2Ma^2(t)] - \mathbf{p}\cdot\mathbf{k}/[Ma^2(t)]$. Under this approximation, introducing spherical coordinates and integrating the trivial angular coordinate, we get:

$$\begin{aligned}\mathrm{Im}\,\bar{\Sigma}_{\mathrm{R}}(t_1,t_2;\mathbf{p}) = &-\int \mathrm{d}\Delta\, e^{i\omega\Delta}\, \frac{g^2 M}{32\pi^2 a(t_1)a(t_2)} \int_0^\infty k\mathrm{d}k \int_{-1}^1 \mathrm{d}x \\ &\times \left[1 - \frac{1}{2M^2}\left(\frac{p^2}{2a^2(t_1)} + \frac{p^2}{2a^2(t_2)} - \frac{pkx}{a^2(t_1)} - \frac{pkx}{a^2(t_2)}\right)\right] \\ &\times e^{-i\int_{t_2}^{t_1}\mathrm{d}t'\,\{M+p^2/[2Ma^2(t')]-kpx/[Ma^2(t')]\}} \\ &\times \left[e^{-i\int_{t_2}^{t_1}\mathrm{d}t'\,k/a(t')}[1+n_\theta(k)] + e^{i\int_{t_2}^{t_1}\mathrm{d}t'\,k/a(t')}\,n_\theta(k)\right], \end{aligned} \tag{20}$$

where $k = |\mathbf{k}|$, $p = |\mathbf{p}|$ and $x = \mathbf{p}\cdot\mathbf{k}/(kp)$. To compute this expression we have to choose a specific model for the evolution of the scale factor $a(t)$. This is what we do in next subsection.

4.2 Linear Approximation to the Scale Factor

As a first step, we approximate the evolution of the scale factor by a linear expansion:

$$a(t) \approx a(T)[1 + H(T)(t-T)]. \tag{21}$$

We keep all terms linear in H and drop terms which contain a higher power in H.

For this approximation to be valid, one should consider timescales much shorter than the inverse Hubble rate. At finite temperature, the relevant timescales appearing in the Feynman diagrams are of the order of the inverse physical temperature. Therefore, the above expression for the scale factor is appropriate when considering physical temperatures which are much larger than the expansion rate (but still much smaller than the fields masses). For the remaining of this subsection the scale hierarchy $M, m \gg \Delta m, \theta/a \gg H$ will be assumed.

The imaginary part of the self-energy is then given by:

$$\begin{aligned}\mathrm{Im}\,\bar{\Sigma}_{\mathrm{R}}(\omega,T;\mathbf{p}) = &-\frac{g^2 M}{32\pi^2 a^2(T)} \\ &\times \int \mathrm{d}\Delta\, e^{i\omega\Delta} \int_0^\infty \mathrm{d}k\,k \int_{-1}^1 \mathrm{d}x \left(1 - \frac{p^2}{2M^2a^2(T)} + \frac{pkx}{M^2a^2(T)}\right) \\ &\times e^{-i\{M+p^2/[2Ma^2(T)]-kpx/[Ma^2(T)]\Delta}\, e^{ik\Delta/a(T)}\, n_\theta(k). \end{aligned} \tag{22}$$

One gets

$$\begin{aligned}\mathrm{Im}\,\bar{\Sigma}_{\mathrm{R}}(\omega,T;\mathbf{p}) = &-\frac{g^2}{16\pi}\int_{-1}^1 \mathrm{d}x\, M\left(M + \frac{p^2}{2Ma^2(T)} - \omega\right) d(p,x) \\ &\times n_\theta\left(a(T)\left[M + \frac{p^2}{2Ma^2(T)} - \omega\right] d(p,x)\right), \end{aligned} \tag{23}$$

where $d(p,x)$ is the Doppler factor

$$d(p,x) := 1 + \frac{px}{Ma(T)}. \tag{24}$$

It is important to remark that the linear dependence on the Hubble parameter H cancels out, so that the result only depends on the scale factor evaluated at time T at this level of approximation. This directly follows from the odd character of $\operatorname{Im}\Sigma_{\mathrm{R}}(t_1,t_2;\mathbf{p})$ under the exchange of t_1 and t_2. The first corrections to the above expression are thus quadratic in H, in the units of M, Δm or $\theta/a(T)$.

In the limit in which the atoms are at rest the expression further simplifies to

$$\operatorname{Im}\bar{\Sigma}_{\mathrm{R}}(\omega,T;\mathbf{0}) = -\frac{g^2}{8\pi}M(M-\omega)n_{\theta/a(T)}(M-\omega). \tag{25}$$

Evaluating the result on-shell one recovers the flat space-time results [3] with a time-dependent temperature:

$$\begin{aligned}\operatorname{Im}\bar{\Sigma}_{\mathrm{R}}(m,T;\mathbf{0}) &= -\frac{g^2}{8\pi}M\Delta m n_{\theta/a(T)}(\Delta m),\\ \Gamma = -\frac{1}{m}\operatorname{Im}\bar{\Sigma}_{\mathrm{R}}(m,T;\mathbf{0}) &= \frac{g^2}{8\pi}\Delta m n_{\theta/a(T)}(\Delta m),\end{aligned} \tag{26}$$

where Γ is the (M-independent) decay rate in cosmological time.

In fact, it can be easily shown that under this linear approximation in H all expressions are given by the corresponding flat spacetime result evaluated at the mid time T. For instance the free propagator becomes

$$\begin{aligned}\bar{G}^{(0)}_{\mathrm{R}}(t_1,t_2;\mathbf{p}) &= \frac{-i}{\sqrt{E_{\mathbf{p}}(t_1)E_{\mathbf{p}}(t_2)}}\sin\left(\int_{t_2}^{t_1}\mathrm{d}t'\,E_{\mathbf{p}}(t')\right)\theta(t_1-t_2)\\ &= \frac{-i}{E_{\mathbf{p}}(T)}\sin[E_{\mathbf{p}}(T)(t_1-t_2)]\theta(t_1-t_2)[1+O(H^2/M^2)].\end{aligned} \tag{27}$$

Similarly, the 'bar' self-energy in an expanding universe is given by the flat-spacetime self-energy by replacing

$$\mathbf{p}\to\mathbf{p}/a(T),\qquad \text{temperature}\to\theta/a(T).$$

This is true as long as the physical temperature $\theta/a(T)$ is larger than the expansion rate H, as explained above. From the flat space-time calculation of [3] we thus deduce that the on-shell real part of the retarded self-energy is given by:

$$\operatorname{Re}\Sigma_{\mathrm{R}}(E_{\mathbf{p}},T;\mathbf{p}) = -\frac{g^2}{6}\frac{\theta^2}{a^2(T)}\frac{m}{\Delta m}[1+O(H^2)]. \tag{28}$$

5 Retarded Propagator and Self-Energy in Cosmology

So far we have only computed the self-energy in a frequency representation. We are still missing an interpretation for this quantity in spacetime. In this section we address this problem in two steps: first, in a short-time approximation, and then for arbitrary long time lapses.

5.1 Short-Time Approximation

In expanding universes the propagators are no longer time-translation invariant. We can nevertheless always express the propagator in Fourier transform with respect to the difference variable $\Delta = t - t'$, while keeping the mid point variable $T = (t + t')/2$ constant:

$$\bar{G}_{\rm R}(\omega, T; \mathbf{p}) := \int \mathrm{d}\Delta\, e^{i\omega\Delta}\, \bar{G}_{\rm R}(T + \Delta/2, T - \Delta/2; \mathbf{p}). \tag{29}$$

This relation is analogous to that of the self-energy that we used before. By re-expressing (16) in terms of the Fourier-transformed quantities we obtain the following cumbersome expression:

$$\begin{aligned}\bar{G}_{\rm R}(\omega, T; \mathbf{p}) = \bar{G}^{(0)}_{\rm R}(\omega, T; \mathbf{p})\\ &- i \int \mathrm{d}\Delta\, \mathrm{d}s\, \mathrm{d}s' \int \frac{\mathrm{d}\omega_1}{(2\pi)} \frac{\mathrm{d}\omega_2}{(2\pi)} \frac{\mathrm{d}\omega_3}{(2\pi)}\\ &\times e^{-i\omega_1(T-\Delta/2-s)-i\omega_2(s-s')-i\omega_3(s'-T-\Delta/2)}\\ &\times \bar{G}^{(0)}_{\rm R}\left(\omega_1, \frac{T}{2} + \frac{\Delta}{4} + s; \mathbf{p}\right) \bar{\Sigma}_{\rm R}\left(\omega_2, \frac{s + s'}{2}; \mathbf{p}\right)\\ &\times \bar{G}_{\rm R}\left(\omega_3, \frac{s'}{2} + \frac{T}{2} - \frac{\Delta}{4}; \mathbf{p}\right),\end{aligned} \tag{30}$$

We shall undertake several steps in order to simplify this expression. First, notice that because of the use of retarded propagators, the different times involved in the above relation are ordered as:

$$t = T + \frac{\Delta}{2} \geq s \geq s' \geq T - \frac{\Delta}{2} = t' \tag{31}$$

Notice also that the free, retarded propagators and the self-energy can all be expanded around the time T as

$$\bar{G}_{\rm R}(\omega, T + \delta t; \mathbf{p}) = \bar{G}_{\rm R}(\omega, T; \mathbf{p}) + \delta t \frac{\partial \bar{G}_{\rm R}(\omega, T; \mathbf{p})}{\partial T} + O(\delta t^2). \tag{32}$$

The propagators and self-energies appearing in the second term in the right hand side of (30) are evaluated at times which differ from T by quantities δt which are of order Δ. Therefore, if we are able to establish that

$$\Delta \frac{\partial \ln \bar{G}_{\rm R}}{\partial T}, \Delta \frac{\partial \ln \bar{G}^{(0)}_{\rm R}}{\partial T}, \Delta \frac{\partial \ln \bar{\Sigma}_{\rm R}}{\partial T} \ll 1, \tag{33}$$

then we can approximate the last line of (30) by evaluating the three functions at time T. Under this assumption the solution to this equation greatly simplifies:

$$\bar{G}_{\rm R}(\omega, T; \mathbf{p}) = \frac{-i}{[-i\bar{G}^{(0)}(\omega, T; \mathbf{p})]^{-1} + \bar{\Sigma}_{\rm R}(\omega, T; \mathbf{p})}, \tag{34}$$

where the free propagator is here approximated by

$$[-i\bar{G}^{(0)}(\omega, T; \mathbf{p})]^{-1} \approx -\omega^2 + \frac{\mathbf{p}^2}{a^2(T)} + m^2. \tag{35}$$

Thus, in order to check the validity of (34) it is sufficient to check the relations (33). By differentiating the above equation, the variation of the free propagator can be approximated by:

$$\Delta \frac{\partial \ln \bar{G}^{(0)}_{\mathrm{R}}}{\partial T} \sim \frac{\mathbf{p}^2 \Delta}{a^2(T) E_{\mathbf{p}}} H \Delta \ll 1, \tag{36}$$

where we have used the time-energy uncertainty relation, $\omega - E_{\mathbf{p}} \sim 1/\Delta$. If we consider time differences Δ much smaller than the typical expansion timescale H^{-1}, and momenta $\mathbf{p}$ which are at most of the order of this inverse time (a weak constraint), this relation is indeed verified. For the self-energy in a thermal bath one obtains

$$\Delta \frac{\partial \ln \bar{\Sigma}_{\mathrm{R}}}{\partial T} \sim \frac{\Delta m}{\theta / a} H \Delta \ll 1, \tag{37}$$

which is also verified if $H^{-1} \gg \Delta$. Finally, since the interacting propagator is obtained from the free propagator and the self-energy, it also verifies the relation when the two others do.

It is important to stress that (33) is stronger than the adiabatic approximation, since the latter only requires the masses to be much larger than the expansion rate. We will hereafter refer to (33) as the short-time approximation.

Under this short-time assumption, one can find a time representation of the propagator by Fourier-transforming equation (34). In order to proceed analytically, we have to further approximate the self-energy by its value at the pole.[2] Assuming a small decay rate, one finds

$$\bar{G}_{\mathrm{R}}(t, t'; \mathbf{p}) = \frac{-i}{R_{\mathbf{p}}(T)} \sin[R_{\mathbf{p}}(T)(t - t')]\, e^{-\Gamma_{\mathbf{p}}(T)(t-t')/2}\, \theta(t - t'). \tag{38}$$

with

$$R^2_{\mathbf{p}}(T) := m^2 + \frac{\mathbf{p}^2}{a^2(T)} + \mathrm{Re}\, \bar{\Sigma}_{\mathrm{R}}(E_{\mathbf{p}}, T; \mathbf{p}) \tag{39}$$

and

$$\Gamma_{\mathbf{p}}(T) := -\frac{1}{R_{\mathbf{p}}(T)} \mathrm{Im}\, \bar{\Sigma}_{\mathrm{R}}(E_{\mathbf{p}}, T; \mathbf{p}). \tag{40}$$

Therefore one recovers the usual interpretation, in which the real part of the self-energy corresponds to the mass shift, and in which the imaginary part corresponds to the decay rate. Notice that both quantities will in general depend on time.

Notice that by the hypothesis of having a very massive field, $R_{\mathbf{p}}(t)$ is much larger than $\Gamma_{\mathbf{p}}(t)$, so the result corresponds to the underdamped solution of the harmonic oscillator. Notice also that the corrections to the energy, given by $R_{\mathbf{p}}(t) - E_{\mathbf{p}}(t)$, can be either positive or negative; the damping rate $\Gamma_{\mathbf{p}}$ is always positive.

5.2 Extension to Arbitrary Large Time Lapses

We now wish to extend the above interpretation to arbitrary time lapses, provided we remain in the adiabatic regime. In order to do so, we will use a different technique. By acting with

[2] This is the usual procedure in field theory in order to deduce the exponential decay law. See for instance [9].

the differential operator

$$\frac{1}{a^3(t)}\frac{\partial}{\partial t}\left(a^3(t)\frac{\partial}{\partial t}\right)+m^2+\xi R(t)+\frac{\mathbf{p}^2}{a^2(t)}$$

on equation (16), we get the equation of motion for the retarded propagator:

$$\begin{aligned}&\left[\frac{1}{a^3(t)}\frac{\partial}{\partial t}\left(a^3(t)\frac{\partial}{\partial t}\right)+m^2+\xi R(t)+\frac{\mathbf{p}^2}{a^2(t)}\right]\left[\frac{\bar{G}_{\mathrm{R}}(t,t';\mathbf{p})}{a^{3/2}(t)a^{3/2}(t')}\right]\\&\quad+\frac{1}{a^{3/2}(t)a^{3/2}(t')}\int \mathrm{d}s\,\bar{\Sigma}_{\mathrm{R}}(t,s;\mathbf{p})\bar{G}_{\mathrm{R}}(s,t';\mathbf{p})=\frac{-i}{a^3(t)}\delta(t-t')\end{aligned}\tag{41}$$

This equation is exact and does not rely on the adiabatic or the short-time approximations. Our aim in this subsection is to find the WKB-like solution to this equation. Since we are interested in a first order adiabatic solution, we start by discarding all terms of the equation of motion which are of higher order in H/M. We thus have

$$\left[\frac{\partial^2}{\partial t^2}+m^2+\frac{\mathbf{p}^2}{a^2(t)}\right]\bar{G}_{\mathrm{R}}(t,t';\mathbf{p})+\int \mathrm{d}s\,\bar{\Sigma}_{\mathrm{R}}(t,s;\mathbf{p})\bar{G}_{\mathrm{R}}(s,t';\mathbf{p})=-i\delta(t-t').\tag{42}$$

Let us now concentrate on the non-local term:

$$\int \mathrm{d}s\,\bar{\Sigma}_{\mathrm{R}}(t,s;\mathbf{p})\bar{G}_{\mathrm{R}}(s,t';\mathbf{p}).$$

We shall replace it by the dominant terms when expressed as a local series. We start by introducing a frequency representation for the propagator and the self-energy:

$$\begin{aligned}\int \mathrm{d}s\,\bar{\Sigma}_{\mathrm{R}}(t,s;\mathbf{p})\bar{G}_{\mathrm{R}}(s,t';\mathbf{p})&=\int \mathrm{d}s\int\frac{\mathrm{d}\omega}{(2\pi)}\frac{\mathrm{d}\omega'}{(2\pi)}\,e^{-i\omega t+i(\omega-\omega')s+i\omega' t'}\\&\quad\times\bar{\Sigma}_{\mathrm{R}}\left(\omega,\frac{t+s}{2};\mathbf{p}\right)\bar{G}_{\mathrm{R}}\left(\omega',\frac{s+t'}{2};\mathbf{p}\right).\end{aligned}\tag{43}$$

In Fourier representation, the retarded propagator becomes large near the mass shell at $\omega'\approx\pm E_{\mathbf{p}}$, so the main contribution to the integral over ω' takes place around this value. On the other hand, we are interested in considering time lapses much longer than the inverse energy. For arbitrary values of ω the exponential $e^{i(\omega-\omega')s}$ becomes rapidly oscillating, and therefore gives a very small contribution when integrated over s. Only in the case $\omega\approx\omega'$ the integral over s becomes large. Replacing ω by ω' in the self-energy, and integrating over ω, we obtain:

$$\begin{aligned}&\int \mathrm{d}s\,\bar{\Sigma}_{\mathrm{R}}(t,s;\mathbf{p})\bar{G}_{\mathrm{R}}(s,t';\mathbf{p})\\&\quad\approx\int \mathrm{d}s\int\frac{\mathrm{d}\omega'}{(2\pi)}\,e^{-i\omega'(t-t')}\,\bar{\Sigma}_{\mathrm{R}}\left(\omega',\frac{t+s}{2};\mathbf{p}\right)\bar{G}_{\mathrm{R}}\left(\omega',\frac{s+t'}{2};\mathbf{p}\right)\delta(t-s)\\&\quad\approx\int\frac{\mathrm{d}\omega'}{(2\pi)}\,e^{-i\omega'(t-t')}\,\bar{\Sigma}_{\mathrm{R}}(\omega',t;\mathbf{p})\bar{G}_{\mathrm{R}}(\omega',T;\mathbf{p}).\end{aligned}\tag{44}$$

As we have already mentioned, the main contribution to the above integral is given by its contribution near the mass shell. Thus, one may replace the self-energy by two terms. Taking into account that the real and imaginary parts of the self-energy in Fourier space are respectively even and odd in ω, we replace the real part of the self-energy by an ω-independent quantity,

$$\mathrm{Re}\,\bar{\Sigma}_{\mathrm{R}}(\omega, t; \mathbf{p}) \to \delta R_{\mathbf{p}}^2(t) := \mathrm{Re}\,\bar{\Sigma}_{\mathrm{R}}(E_{\mathbf{p}}, t; \mathbf{p}), \tag{45}$$

and the imaginary part by a linear function in ω,

$$\mathrm{Im}\,\bar{\Sigma}_{\mathrm{R}}(\omega, t; \mathbf{p}) \to -\omega \Gamma_{\mathbf{p}}(t), \tag{46}$$

where $\Gamma_{\mathbf{p}}(t)$ is given by (40). Then, the non-local term of the equation of motion can be approximated as

$$\int \mathrm{d}s\, \bar{\Sigma}_{\mathrm{R}}(t, s; \mathbf{p}) \bar{G}_{\mathrm{R}}(s, t'; \mathbf{p}) \approx \int \frac{\mathrm{d}\omega}{(2\pi)} e^{-i\omega(t-t')} [\delta R_{\mathbf{p}}^2(t) - i\omega \Gamma_{\mathbf{p}}(t)] \bar{G}_{\mathrm{R}}(\omega, T; \mathbf{p}) \tag{47}$$

Going back to time space this term can be expressed as:

$$\int \mathrm{d}s\, \bar{\Sigma}_{\mathrm{R}}(t, s; \mathbf{p}) \bar{G}_{\mathrm{R}}(s, t'; \mathbf{p}) \approx \left[\delta R_{\mathbf{p}}^2(t) + \Gamma_{\mathbf{p}}(t) \frac{\partial}{\partial t} \right] \bar{G}_{\mathrm{R}}(t, t'; \mathbf{p}). \tag{48}$$

Therefore, the equation of motion (41) can be approximated by:

$$\left[\frac{\partial^2}{\partial t^2} + \Gamma_{\mathbf{p}}(t) \frac{\partial}{\partial t} + R_{\mathbf{p}}^2(t) \right] \bar{G}_{\mathrm{R}}(t, t'; \mathbf{p}) = -i\delta(t - t') \tag{49}$$

where $R_{\mathbf{p}}(t)$ is given by (39). We thus get that, in the presence of interactions, the Green function at fixed conformal momentum corresponds to that of a damped harmonic oscillator with time-dependent coefficients.

There is a slightly different way to get (48). The left-hand side of this equation can always be formally expanded as a series of local terms:

$$\int \mathrm{d}s\, \bar{\Sigma}_{\mathrm{R}}(t, s; \mathbf{p}) \bar{G}_{\mathrm{R}}(s, t'; \mathbf{p}) = \sum_{n=0}^{\infty} a_n(t) \frac{\partial^n}{\partial t^n} \bar{G}_{\mathrm{R}}(t, t'; \mathbf{p}). \tag{50}$$

The more adiabatic the theory is (i.e., the smaller is the ratio H/M), the more dominant the terms with fewer derivatives are. Even terms in this expansion are related to reversible effects (mass shift and dispersion), whereas odd terms are related to dissipative effects. Therefore, the dominant even term is a mass squared shift, $a_0(t) = \delta R_{\mathbf{p}}^2(t)$, and the dominant odd term is the decay rate $a_1(t) = \Gamma_{\mathbf{p}}(t)$. One can consider (48) as defining these two quantities. In this viewpoint, one still needs to show that the shift $\delta R_{\mathbf{p}}^2(t)$ and the width $\Gamma_{\mathbf{p}}(t)$ are approximately given by the Fourier transform of the self-energy evaluated on-shell, as indicated in (40) and (45). That this is the case is the important result of this section.

We can now find a WKB solution to (49) in a similar way as in Sect. 3. By considering an ansatz analogous to that of (10),

$$\bar{G}_{\mathrm{R}}(u, u'; \mathbf{p}) = \frac{-i}{\sqrt{-\partial_u A(u, u'; \mathbf{p}) \partial_{u'} A(u, u'; \mathbf{p})}} \sin\left(\frac{A(u, u'; \mathbf{p})}{h} \right) \times e^{-B(u,u';\mathbf{p})/h} \theta(u - u'), \tag{51}$$

one finds that the adiabatic solution reads

$$\bar{G}_{\mathrm{R}}(t_1, t_2; \mathbf{p}) = \frac{-i}{\sqrt{R_{\mathbf{p}}(t_1) R_{\mathbf{p}}(t_2)}} \sin\left(\int_{t_2}^{t_1} \mathrm{d}t'\, R_{\mathbf{p}}(t') \right) e^{-\int_{t_2}^{t_1} \mathrm{d}t'\, \Gamma_k(t')/2}\, \theta(t_1 - t_2) + O(H^2, g^4, Hg^2). \tag{52}$$

Going to the short time limit one recovers (38), as expected.

To conclude this section, it is interesting to notice that (42) and (50) can be viewed as defining an effective dispersion relation. Even though the notion of modes no longer exists in the presence of interactions, the two point Green function always exists and always obeys a linear differential equation, which governs the effective propagation and therefore *defines* the dispersion relation [28].

6 Summary and Discussion

The goal of this paper is to analyze the quantum effects in the propagation of interacting fields in a cosmological background. This issue may play an important role in justifying the non-trivial dispersion relations which have been used when addressing the trans-Planckian question in the context of black holes [5, 16, 30, 31] and cosmology [20–23]. Interactions could indeed significantly modify the field propagation when approaching the event horizon of a black hole [6, 25–27] or at primordial stages of inflation [2]. In order to further investigate this possibility, we have analyzed a particular model consisting of a doublet of massive particles propagating in an isotropic and homogeneous expanding universe filled with radiation. In this model, the propagation of massive particles is indeed governed by an effective dispersion relation which contains an absorptive, the origin of which being the imaginary part of the self-energy. More technically, we have considered the following aspects.

First, we have adopted an appropriate theoretical framework to consider interacting quantum field theory in curved spacetimes, and to be able to extract the relevant information from the self-energy of the fields. To this end, the masses of the fields were assumed to be much larger than the expansion rate of the universe. This was a key assumption, because it allowed to introduce the adiabatic (WKB) approximation, which not only makes the problem solvable, but also allows having a well-defined particle concept even in absence of asymptotic regimes. In order to have interesting dynamics, the mass gap between the two massive states was taken to be much lower than the masses of the fields.

Second, to extract the dissipative effects, we have computed the imaginary part of the self-energy, when the massless field is at finite temperature. Since translation invariance is broken in an expanding universe, we computed the Feynman diagrams in a time representation. However, to extract the physical information from the real and imaginary parts of the self-energy, we introduced a frequency representation around the mid time, in close analogy to what is done in [10], [7], and [1]. While the calculation was presented in full generality, results were only reached under the assumption of large temperatures and mass gaps compared to the expansion rate. In this regime, the transition rates only depend on the local temperature (since the linear terms in the Hubble parameter cancel out). One thus recover the flat-spacetime thermal decay rates with a time- (or scale-) dependent temperature. In the regime in which the expansion rate becomes comparable to the mass gap, one expects non-adiabatic transitions. We plan to study these transitions in a separate paper [4].

Third, for short time lapses (compared to the typical expansion time) the Fourier transform around the semisum time is easily justified, and one recovers an expression analogous to that of flat space-time, with real and imaginary parts of the self-energy. However, the adiabatic approximation of having no spontaneous creation of pairs of massive quanta is valid for arbitrary long times. It is therefore important to study the extension of the above analysis to long time intervals. In this case the frequency representation of the propagator is no longer useful, so we worked directly with the equation of motion of the retarded propagator. The radiative corrections induce a non-local term in this equation. We approximated the even and the odd parts of this term by its dominant contribution in a temporal gradient expansion. The resulting equation was solved by means of the WKB method. The solution, (52), shows that the long-time behavior of the propagator can be computed as an integration over radiative effects which are evaluated in the short-time lapse approximation we first studied.

In conclusion, the decay rate, derived from the imaginary part of the self-energy, has a secular character, as expected. Even small decay rates could thus give an important effect when integrated over large periods of time. Moreover, the temperature increases as one goes backwards in time in an expanding universe. Therefore, if one considers remote times or large time lapses, the value of the propagator will generically decrease. This would imply vanishing correlations with early configurations. If a similar phenomenon also applies to near horizon black hole propagation, the description of black hole radiance would radically differ from the usual free field description as it would instead be similar to that represented in Fig. 5 of [8].

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